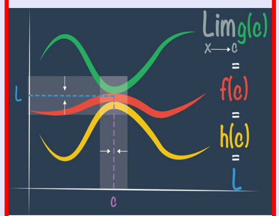
Calculus I

Lecture 9



Feb 19-8:47 AM

Differentiation Formulas

1) $\frac{d}{dx}[c]=0$

Constant

ex: $\frac{d}{dx}[5]=0$, $\frac{d}{dx}[\pi]=0$, $\frac{d}{dx}[t^2]=0$

Respect to x

2) $\frac{d}{dx}[x]=1$

$\frac{dx}{dx}=1$

3) $\frac{d}{dx}[x^n]=nx^{n-1}$

$\frac{d}{dx}[x^6]=6x^{6-1}$

$=6x^5$

, $\frac{d}{dx}[\sqrt[5]{x}]=\frac{d}{dx}[x^{1/5}]$

$=\frac{1}{5}x^{\frac{1}{5}-1}$

$=\frac{1}{5}x^{-4/5}=\frac{1}{5x^{4/5}}$

$=\frac{1}{5\sqrt[5]{x^4}}$

$\frac{1}{5\sqrt[5]{x^4}}\cdot\frac{\sqrt[5]{x}}{\sqrt[5]{x}}$

$=\frac{\sqrt[5]{x}}{5\sqrt[5]{x^5}}=\frac{\sqrt[5]{x}}{5x}$

Jul 1-8:08 AM

4) $\frac{d}{dx}[c f(x)] = c \frac{d}{dx}[f(x)]$

$$\frac{d}{dx}\left[\frac{1}{2}x^2\right] = \frac{1}{2}\frac{d}{dx}[x^2]$$

$$= \frac{1}{2}\cdot 2x^{2-1} = \frac{1}{2}\cdot 2x^1 = \boxed{x}$$

$$\frac{d}{dx}\left[3\sqrt[3]{x^2}\right] = 3\frac{d}{dx}\left[x^{2/3}\right] = \cancel{3}\cdot\frac{2}{\cancel{3}}x^{\frac{2}{3}-1}$$

$$= 2x^{-1/3} = \frac{2}{x^{1/3}} = \boxed{\frac{2}{\sqrt[3]{x}}}$$

5) $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

$$\frac{d}{dx}[x^3 - 10] = \frac{d}{dx}[x^3] - \frac{d}{dx}[10]$$

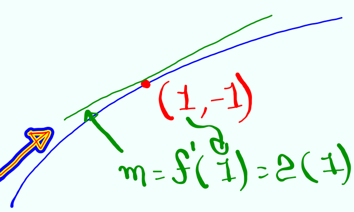
$$= 3x^2 - 0 = \boxed{3x^2}$$

Jul 1-8:16 AM

Find eqn of the $\boxed{\text{tan. line}}$ to the graph of

$f(x) = x^2 + 4x - 6$ $\boxed{\text{at } x = 1}$

$f(x) = x^2 + 4x - 6$



$f(1) = 1^2 + 4(1) - 6$
$= -1$

$m = f'(1) = 2(1) + 4 = 6$

$$f'(x) = \frac{d}{dx}[x^2 + 4x - 6] = \frac{d}{dx}[x^2] + \frac{d}{dx}[4x] - \frac{d}{dx}[6]$$

$$= 2x + 4\cdot 1 - 0$$

$$= \boxed{2x + 4}$$

$y - y_1 = m(x - x_1)$

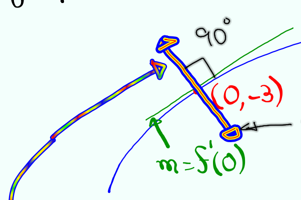
$y - (-1) = 6(x - 1)$

$y + 1 = 6x - 6$

$\boxed{y = 6x - 7}$

Jul 1-8:25 AM

Find the eqn of the normal line to the graph of $f(x) = 4x + \sin x - 3$ at $x=0$.



$f(x) = 4x + \sin x - 3$

$f(0) = \cancel{4(0)}^{0} + \cancel{\sin 0}^{0} - 3$

$= -3$

$m = \dfrac{-1}{f'(0)} = \boxed{\dfrac{-1}{5}}$

$f(x) = 4x + \sin x - 3$

$f'(x) = \frac{d}{dx}[4x] + \frac{d}{dx}[\sin x] - \frac{d}{dx}[3] = 4 + \cos x - 0$

$= 4 + \cos x$

$f'(0) = 4 + \cancel{\cos 0}^{1} = \boxed{5}$

$y - y_1 = m(x - x_1)$

$y - (-3) = \frac{-1}{5}(x - 0)$

$y + 3 = \frac{-1}{5}x$

$\boxed{y = \frac{-1}{5}x - 3}$

Jul 1-8:31 AM

6) $\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

Solid Product

Product Rule

$\frac{d}{dx}[x^2 \tan x]$

$= \frac{d}{dx}[x^2] \cdot \tan x + x^2 \cdot \frac{d}{dx}[\tan x]$

$= \boxed{2x \tan x + x^2 \cdot \sec^2 x}$

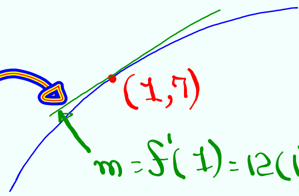
$\frac{d}{dx}\left[\boxed{\sin x}\,\boxed{\cos x}\right] = \cos x \cos x + \sin x(-\sin x)$

$= \cos^2 x - \sin^2 x$

$= \boxed{\cos 2x}$

Jul 1-8:39 AM

Find eqn of the tan. line to the graph of $f(x) = (2x-1)(3x+4)$ at $x=1$.



$f(x) = (2x-1)(3x+4)$

$f(1) = (2\cdot 1 - 1)(3\cdot 1 + 4)$
$= \boxed{7}$

$m = f'(1) = 12(1) + 5$
$= \boxed{17}$

$y - y_1 = m(x - x_1)$

$y - 7 = 17(x-1)$

$\boxed{y = 17x - 10}$

$f(x) = (2x-1)(3x+4)$

$f'(x) = \frac{d}{dx}\left[(2x-1)(3x+4)\right]$

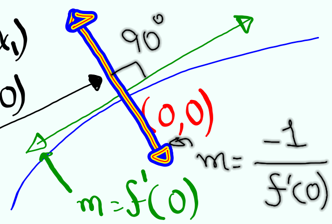
$= \underbrace{\frac{d}{dx}\left[2x-1\right]}\cdot(3x+4) + (2x-1)\cdot\frac{d}{dx}\left[3x+4\right]$

$= 2(3x+4) + (2x-1)\cdot 3$

$= 6x + 8 + 6x - 3 = \boxed{12x + 5}$

Jul 1-8:47 AM

Find the eqn of the normal line to the graph of $f(x) = (2x+1)\cdot\tan x$ at $x=0$.



$f(x) = (2x+1)\cdot\tan x$

$y - y_1 = m(x - x_1)$

$y - 0 = -1(x - 0)$

$\boxed{y = -x}$

$f(0) = (2\cdot 0 + 1)\cdot\cancel{\tan 0}^{\,0}$

$= 1\cdot 0 = 0$

$m = f'(0)$

$m = \frac{-1}{f'(0)} = \frac{-1}{1}$
$= \boxed{-1}$

$f(x) = (2x+1)\cdot\tan x$

$f'(x) = 2\cdot\tan x + (2x+1)\cdot\sec^2 x$

$f'(0) = 2\cancel{\tan 0}^{\,0} + (2\cdot 0 + 1)\cdot\cancel{\sec^2 0}$

$= 2\cdot 0 + 1\cdot 1^2$

$= 1$

$\cos 0 = 1$

$\sec 0 = \frac{1}{\cos 0}$
$= 1$

Jul 1-8:55 AM

find $\frac{d}{dx}\left[(x^2+3)(x^2-3)\right]$

Method I:

$$\frac{d}{dx}\left[(\overset{A}{x^2}\overset{+B}{+3})(\overset{A-B}{x^2-3})\right] = \frac{d}{dx}\left[\overset{A^2-B^2}{(x^2)^2-3^2}\right]$$

$$= \frac{d}{dx}\left[x^4-9\right] = 4x^3 - 0 = \boxed{4x^3}$$

Method II:

$$\frac{d}{dx}\left[(x^2+3)(x^2-3)\right] = 2x\cdot(x^2-3) + (x^2+3)\cdot 2x$$

$$= 2x^3 \cancel{-6x} + 2x^3 \cancel{+6x} = \boxed{4x^3}$$

Jul 1-9:05 AM

find the $\underbrace{\text{Second derivative}}_{f''(x)}$ of $\underbrace{\text{any quadratic function.}}_{f(x)=ax^2+bx+c}$

f-double Prime of x

$$f''(x) = \frac{d}{dx}\left[f'(x)\right]$$

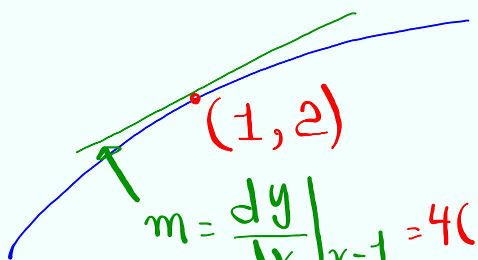
$$f(x) = ax^2+bx+c$$

$$f'(x) = 2ax + b$$

$$\boxed{f''(x) = 2a}$$

Jul 1-9:10 AM

Find eqn of the tan. line to the graph of $y = x^4 + 2x^2 - x$ at $x = 1$.



$y = x^4 + 2x^2 - x$

$y' = 4x^3 + 4x - 1$

$m = \left.\dfrac{dy}{dx}\right|_{x=1} = 4(1)^3 + 4(1) - 1 = \boxed{7}$

$y - 2 = 7(x - 1) \longrightarrow \boxed{y = 7x - 5}$

Jul 1-9:14 AM

7) $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{\frac{d}{dx}[f(x)]\,g(x) - f(x)\cdot\frac{d}{dx}[g(x)]}{[g(x)]^2}$

Quotient Rule

$= \dfrac{f'(x)\,g(x) - f(x)\,g'(x)}{[g(x)]^2}$

$\dfrac{d}{dx}\left[\dfrac{2x}{3x+5}\right] = \dfrac{2(3x+5) - 2x(3)}{(3x+5)^2}$

$= \dfrac{\cancel{6x} + 10 - \cancel{6x}}{(3x+5)^2} = \boxed{\dfrac{10}{(3x+5)^2}}$

$\dfrac{d}{dx}\left[\dfrac{x^2-2}{x^2+2}\right] = \dfrac{2x(x^2+2) - (x^2-2)\cdot 2x}{(x^2+2)^2}$

$= \dfrac{\cancel{2x^3} + 4x - \cancel{2x^3} + 4x}{(x^2+2)^2} = \boxed{\dfrac{8x}{(x^2+2)^2}}$

Jul 1-9:33 AM

Show $\frac{d}{dx}[\tan x] = \sec^2 x$

$$\boxed{\frac{d}{dx}[\tan x]} = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right] = \frac{\overset{f'}{\cos x}\cdot\overset{g}{\cos x} - \overset{f}{\sin x}\cdot\overset{g'}{(-\sin x)}}{[\cos x]^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$= \left[\frac{1}{\cos x}\right]^2 = \boxed{\sec^2 x}$$

Jul 1-9:40 AM

find $f'(x)$ if $f(x) = \frac{x^3}{x^2-1}$

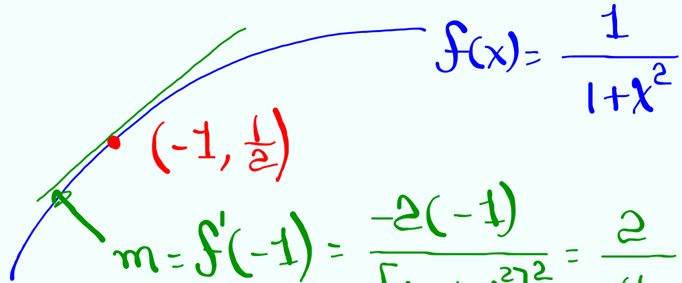
$$f'(x) = \frac{\frac{d}{dx}[x^3]\cdot(x^2-1) - x^3\cdot\frac{d}{dx}[x^2-1]}{(x^2-1)^2}$$

$$= \frac{3x^2(x^2-1) - x^3\cdot 2x}{(x^2-1)^2} = \frac{3x^4-3x^2-2x^4}{(x^2-1)^2}$$

$$= \boxed{\frac{x^4-3x^2}{(x^2-1)^2}}$$

Jul 1-9:45 AM

Find the eqn of the Tan. line to the graph of $f(x)=\dfrac{1}{1+x^2}$ at $x=-1$.



$$m=f'(-1)=\frac{-2(-1)}{[1+(-1)^2]^2}=\frac{2}{4}=\frac{1}{2}$$

$$f(x)=\frac{1}{1+x^2}$$

$$f'(x)=\frac{0\cdot(1+x^2)-1\cdot 2x}{(1+x^2)^2}=\frac{-2x}{(1+x^2)^2}$$

$$y-\frac{1}{2}=\frac{1}{2}(x+1)$$

$$\boxed{y=\frac{1}{2}x+1}$$

Jul 1-9:49 AM

$y=\dfrac{\cos x}{1-\sin x}$, Find $\left.\dfrac{dy}{dx}\right|_{x=0}$

$$y'=\frac{-\sin x\,(1-\sin x)-\cos x\cdot(-\cos x)}{[1-\sin x]^2}$$

$$=\frac{-\sin x+\sin^2 x+\cos^2 x}{[1-\sin x]^2}=\frac{-\sin x+1}{[1-\sin x]^2}$$

$$=\frac{1-\sin x}{[1-\sin x]^2}=\boxed{\frac{1}{1-\sin x}}$$

$$\left.\frac{dy}{dx}\right|_{x=0}=\left.\frac{1}{1-\sin x}\right|_{x=0}=\frac{1}{1-\sin 0}=\boxed{1}$$

Jul 1-9:56 AM

$f(x) = \dfrac{\tan x - 1}{\sec x}$ Find $f'(x)$ and Simplify.

Method I:

$$\frac{d}{dx}\left[\frac{\tan x - 1}{\sec x}\right] = \frac{\sec^2 x \cdot \boxed{\sec x} - (\tan x - 1)\cdot \boxed{\sec x}\tan x}{\sec^2 x}$$

$$= \frac{\cancel{\sec x}\left[\sec^2 x - \tan^2 x + \tan x\right]}{\sec^{\cancel{2}} x}$$

$$= \frac{\cancel{\tan^2 x} + 1 - \cancel{\tan^2 x} + \tan x}{\sec x}$$

$$= \boxed{\frac{1 + \tan x}{\sec x}}$$

Method II:

$$\frac{d}{dx}\left[\frac{\tan x - 1}{\sec x}\right] = \frac{d}{dx}\left[\frac{\frac{\sin x}{\cos x} - 1}{\frac{1}{\cos x}}\right] = \frac{d}{dx}\left[\frac{\sin x - \cos x}{1}\right]$$

LCD = $\cos x$

$$= \frac{d}{dx}\left[\sin x - \cos x\right] = \cos x - (-\sin x)$$

$$= \cos x + \sin x$$

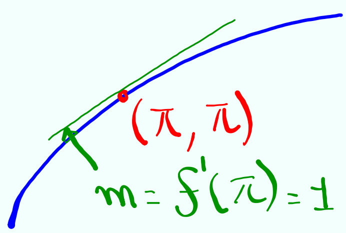
Show $\dfrac{1 + \tan x}{\sec x} = \cos x + \sin x$

$$\frac{1 + \tan x}{\sec x} = \frac{1 + \frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \frac{\cos x + \sin x}{1} = \cos x + \sin x$$

LCD = $\cos x$

Jul 1-10:02 AM

Find the eqn of the tan. line to the graph of $f(x) = x + \tan x$ at $x = \pi$.



$f(x) = x + \tan x$

$f(\pi) = \pi + \tan \pi$ ($\tan \pi \to 0$)

$= \pi$

(π, π)

$m = f'(\pi) = 1 + \sec^2 \pi = 1 + 1$

$= 2$

$\cos \pi = -1$

$\sec \pi = \dfrac{1}{-1}$

$= -1$

$\sec^2 \pi = 1$

$f(x) = x + \tan x$

$f'(x) = 1 + \sec^2 x$

$$\boxed{\begin{aligned} y - y_1 &= m(x - x_1) \\ y - \pi &= 2(x - \pi) \end{aligned}}$$

$$\boxed{y = 2x - \pi}$$

Jul 1-10:19 AM

$f(x) = x^3 - 12x + 2$

1) $f'(x) = \boxed{3x^2 - 12}$

2) $f''(x) = \boxed{6x}$

3) Solve $f'(x) = 0$

Solve $3x^2 - 12 = 0$

$3(x^2 - 4) = 0$

$3(x+2)(x-2) = 0$

$x+2=0$ $\quad$ $x-2=0$

$\boxed{x=-2}$ $\quad$ $\boxed{x=2}$

4) Solve $f''(x) = 0$

Solve $6x = 0$

$\boxed{x=0}$

Jul 1-10:29 AM

$f(x) = \dfrac{x}{x-1}$

1) $f'(x) = \dfrac{1(x-1) - x \cdot 1}{(x-1)^2}$

$= \dfrac{-1}{(x-1)^2} = \dfrac{-1}{x^2-2x+1}$

2) $f''(x) = \dfrac{\cancel{0 \cdot (x^2-2x+1)}^{\,0} - (-1)(2x-2)}{(x^2-2x+1)^2}$

$= \dfrac{2\cancel{(x-1)}}{(x-1)^4} = \dfrac{2}{(x-1)^3}$

3) find x where $f'(x) = 0$ or undefined

$f'(x) = 0 \rightarrow \dfrac{-1}{(x-1)^2} = 0$ No Soln.

$f'(x)$ is undefined at $x=1$

4) find x where $f''(x) = 0$ or undef.

$f''(x) \neq 0$

$f''(x)$ undefined $x = 1$

Jul 1-10:36 AM

find $\frac{d^{20}}{dx^{20}}[\sin x] = \sin x$ 20th derivative

Patterns

$\frac{d}{dx}[\sin x] = \cos x$ $\frac{d^5}{dx^5}[\sin x] = \cos x$ $\frac{d^9}{dx^9}[\sin x]$

$\frac{d^2}{dx^2}[\sin x] = -\sin x$ $\frac{d^6}{dx^6}[\sin x] = -\sin x$

$\frac{d^3}{dx^3}[\sin x] = -\cos x$ ✓

$\frac{d^4}{dx^4}[\sin x] = \sin x$ ✓

Jul 1-10:47 AM

One application of derivative

Linear Approximation

$$f(x) \approx f(a) + f'(a)(x-a)$$

Estimate 10.1^3

1) Using Calc. → $\boxed{1030}.301$

$10.1^3 \approx 10^3$

$f(x) = x^3$

$a = 10$

$f'(x) = 3x^2$

$f(x) \approx f(a) + f'(a)(x-a)$

$x^3 \approx 10^3 + 3 \cdot 10^2(x-10)$

$x^3 \approx 1000 + 300(x-10)$

$10.1^3 \approx 1000 + 300(10.1-10)$

$= 1000 + 300(.1)$

$= 1000 + 30$

$= \boxed{1030}$

Jul 1-10:52 AM

Estimate $\sqrt{10}$

1) By Calc. $\rightarrow$ 3.162

Linear Approx.

$$\underbrace{f(x) \approx f(a) + f'(a)(x-a)}$$

$f(x) = \sqrt{x}$

$a = 9$

$f(a) = \sqrt{9} = 3$

$f'(x) = \dfrac{1}{2\sqrt{x}}$

$f'(9) = \dfrac{1}{2\sqrt{9}} = \dfrac{1}{6}$

$$\sqrt{x} \approx 3 + \frac{1}{6}(x-9)$$

$$\sqrt{10} \approx 3 + \frac{1}{6}(10-9)$$

$$= 3 + \frac{1}{6} \cdot 1$$

$$= 3 + \frac{1}{6} = \boxed{\frac{19}{6}}$$

$$\approx \boxed{3.166}$$

Jul 1-10:59 AM